Universal quantum transport in ultracold atomic gases

How slowly can spins diffuse?

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Strongly interacting Fermi gas

Bloch, Dalibard & Zwerger 2008

- two-component Fermi gas ⬆, ⬇ with contact interaction

$$\mathcal{H} = \int d\mathbf{x} \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger \left(-\frac{\hbar^2\nabla^2}{2m} - \mu_\sigma\right)\psi_\sigma + g_0 \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow$$

- scattering amplitude (3d)

$$f(k) = \frac{1}{-1/a - ik + r_e k^2/2}$$

- strong scattering in unitary limit, scale invar.

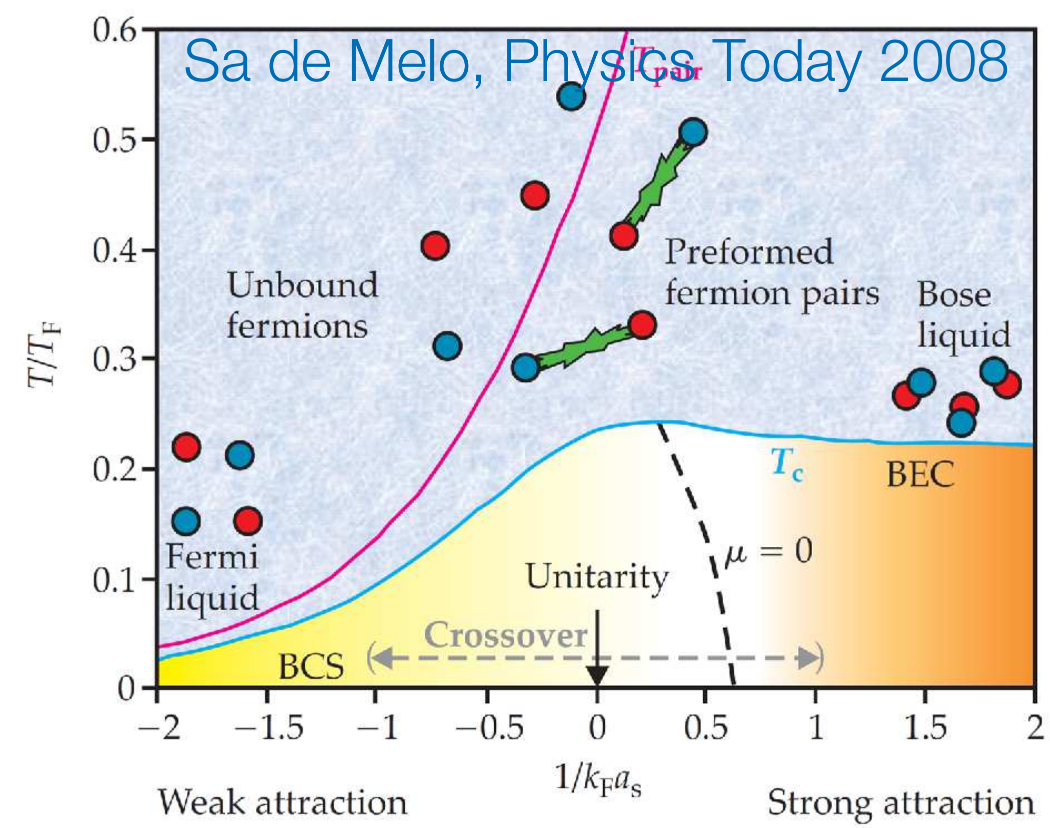
$$1/a = 0: \quad f(k \to 0) = \frac{i}{k}$$

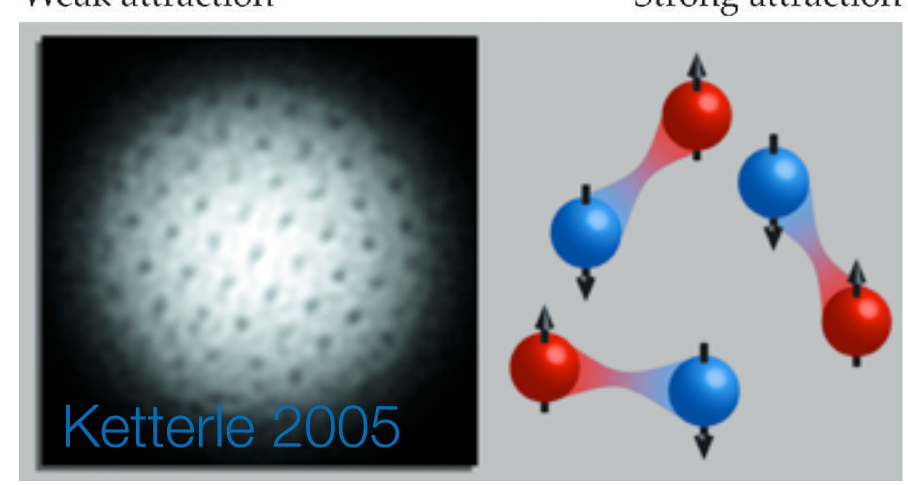
- universal for dilute system

$$r_e \ll n^{-1/3}$$

- superfluid of fermion pairs below

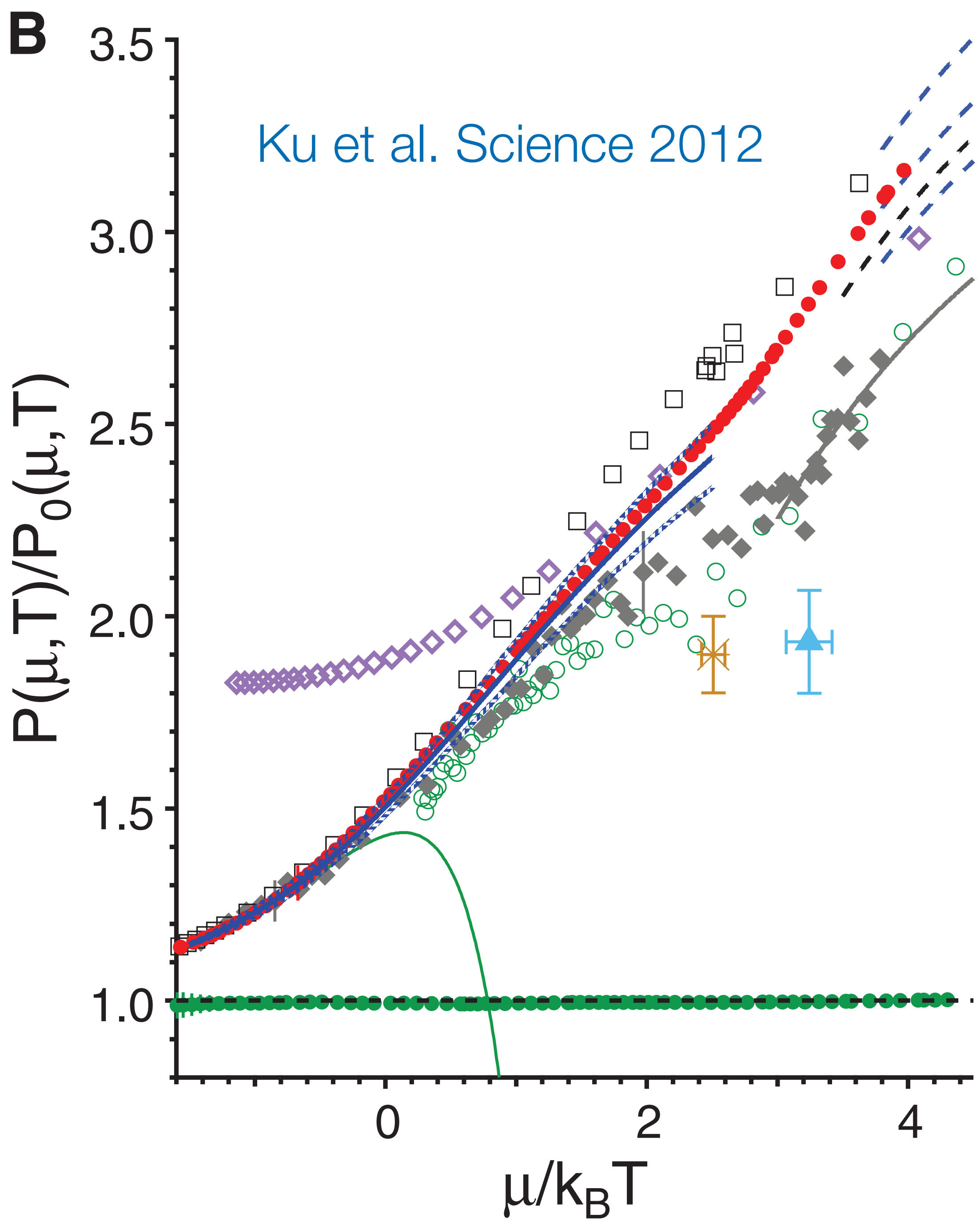
$$T_c/T_F \approx 0.16$$ Ku et al. Science 2012





Pressure equation of state

- universal scaling function P(μ,T)

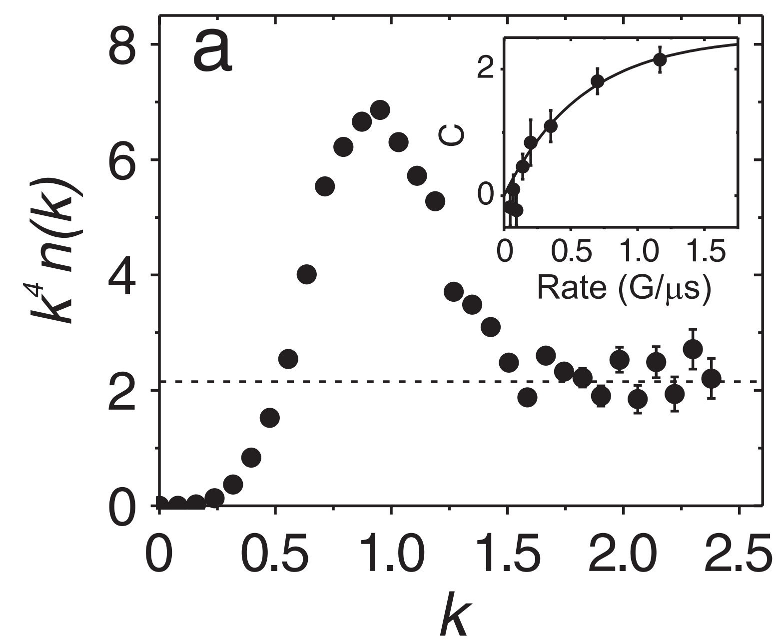


- experiment in Zwierlein group (red):
  Tc=0.167(13), ξ=0.370(5)(8)
  Ku et al. Science 2012; Zürn et al. PRL 2013
- Luttinger-Ward calculation (squares):
  Tc=0.16(1), ξ=0.36(1)
  Haussmann 1993/4; Haussmann et al. PRA 2007
- Bold Diagrammatic Monte Carlo (blue):
  very good agreement in normal phase
  van Houcke et al. Nature Phys. 2012
- Quantum Monte Carlo (green circles):
  Bulgac et al. PRL 2006; Drut et al. PRA 2012
  McNeil Forbes et al. PRL 2011

Contact density

- usually: only low-energy properties universal
- dilute Fermi gas, up to inverse range $k \lesssim r_0^{-1}$

$$n(k) \simeq \frac{C}{k^4}$$



Stewart et al. PRL 2010

- dilute system: **universal short-distance behavior** for $r_0 \lesssim r \lesssim \ell$

many-body **few-body**

$$\langle \hat{n}_\uparrow(r)\hat{n}_\downarrow(0)\rangle \simeq C\left(\frac{1}{r}-\frac{1}{a}\right)^2$$

- **Tan contact density C:** probability of finding ⬆ and ⬇ close together (property of medium) Tan 2008; Braaten and Platter 2008

Contact density

- universal high-energy tails in correlation functions
  intuitively: absorb high-energy perturbation by 2 particles close together
  ➡ absorption rate proportional to **C**

- predictive power (cf. Landau parameters):
  measure one tail, know all tails; Tan adiabatic thm. $$\frac{dP}{d(1/a)} = \frac{C}{4\pi m}$$

van Houcke+ arXiv:1303.6245

- grey crosses: QMC
  Drut et al. PRL 2011

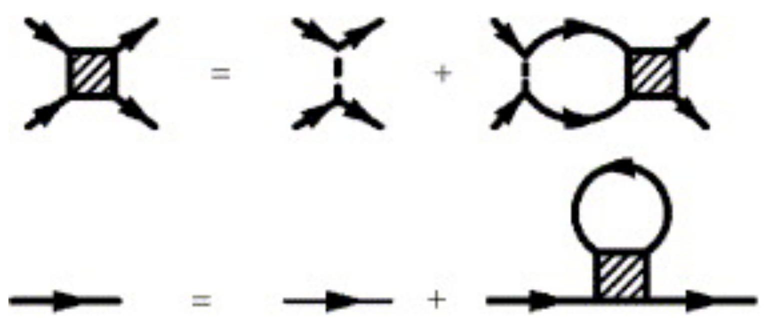
- diamonds: experiment
  Sagi et al. PRL 2012

- red line: Luttinger-Ward
  Enss, Haussmann & Zwerger Ann. Phys. 2011

- blue circles: Bold Diag. MC
  van Houcke et al. arXiv:1303.6245

Luttinger-Ward theory

- Luttinger-Ward (2PI) computation: repeated particle-particle scattering

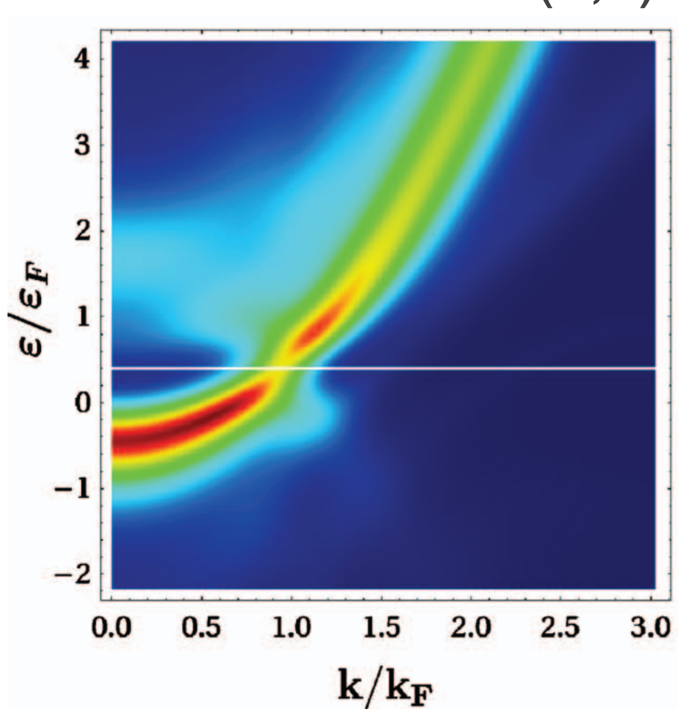


self-consistent T-matrix

Haussmann 1993, 1994;
Haussmann et al. 2007

self-consistent fermion propagator
(300 momenta / 300 Matsubara frequencies)

- spectral function A(k,ε) at Tc



Haussmann et al. 2009

works above and below Tc;
directly in continuum limit

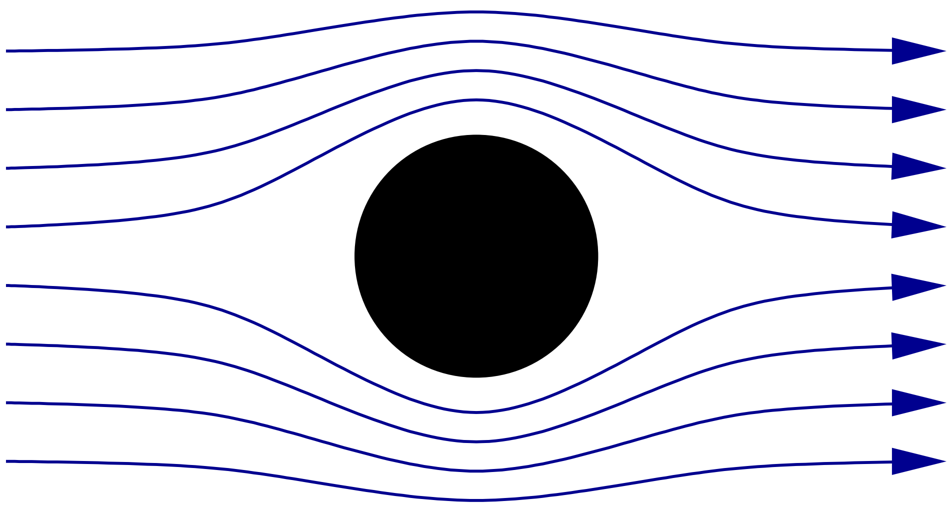
Tc and ξ agree with experiment

conserving: exactly fulfills scale invariance and Tan relations
Enss PRA 2012

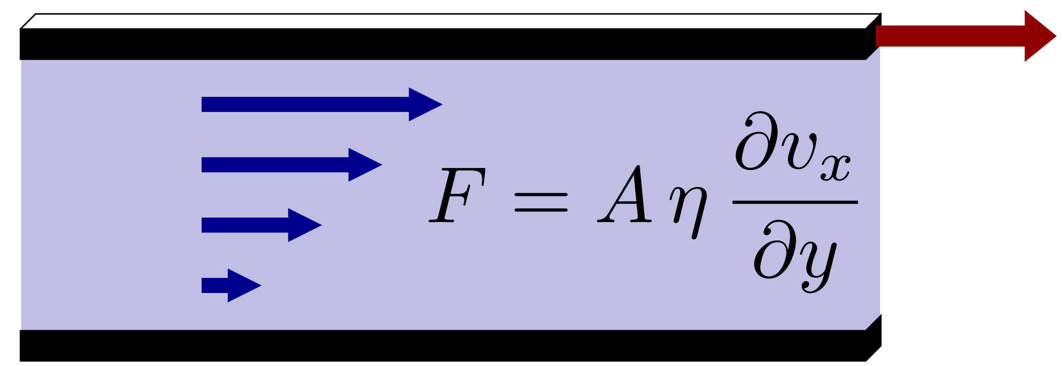
Can mass flow without friction?

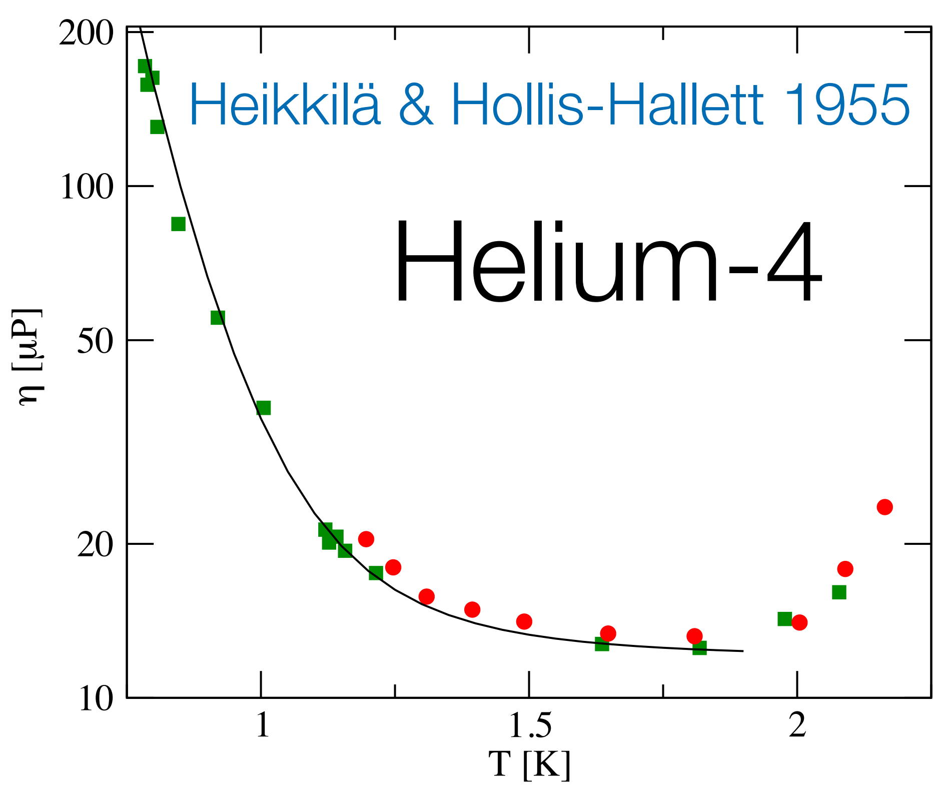
Schäfer, Teaney 2009

- flow without friction?



shear viscosity η:





- kinetic theory suggests:

$$\eta/s \gtrsim \mathcal{O}(1)\,\hbar/k_B$$

- holographic duality:

perfect fluidity $\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$

conjectured as universal lower bound

Kovtun, Son, Starinets 2005

Viscosity in linear response: Kubo formula

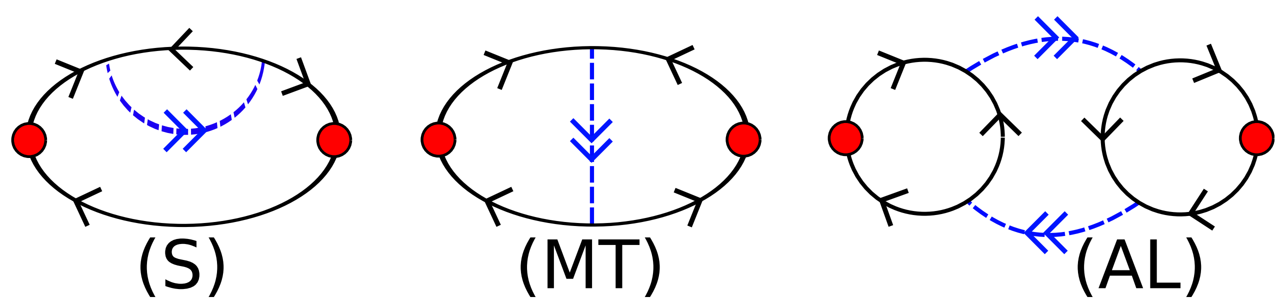
- viscosity from stress correlations (cf. hydrodynamics):

$$\eta(\omega) = \frac{1}{\omega}\,\mathrm{Re}\int_0^\infty dt\, e^{i\omega t}\int d^3x\,\left\langle \left[\hat{\Pi}_{xy}(\boldsymbol{x},t), \hat{\Pi}_{xy}(0,0)\right]\right\rangle$$

with stress tensor $\hat{\Pi}_{xy} = \sum_{\mathbf{p},\sigma} \frac{p_x p_y}{m}\, c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma}$ (cf. Newton $\frac{\partial v_x}{\partial y}$)

- correlation function (Kubo formula): Enss, Haussmann & Zwerger Ann. Phys. 2011

$\eta(\omega)$ =



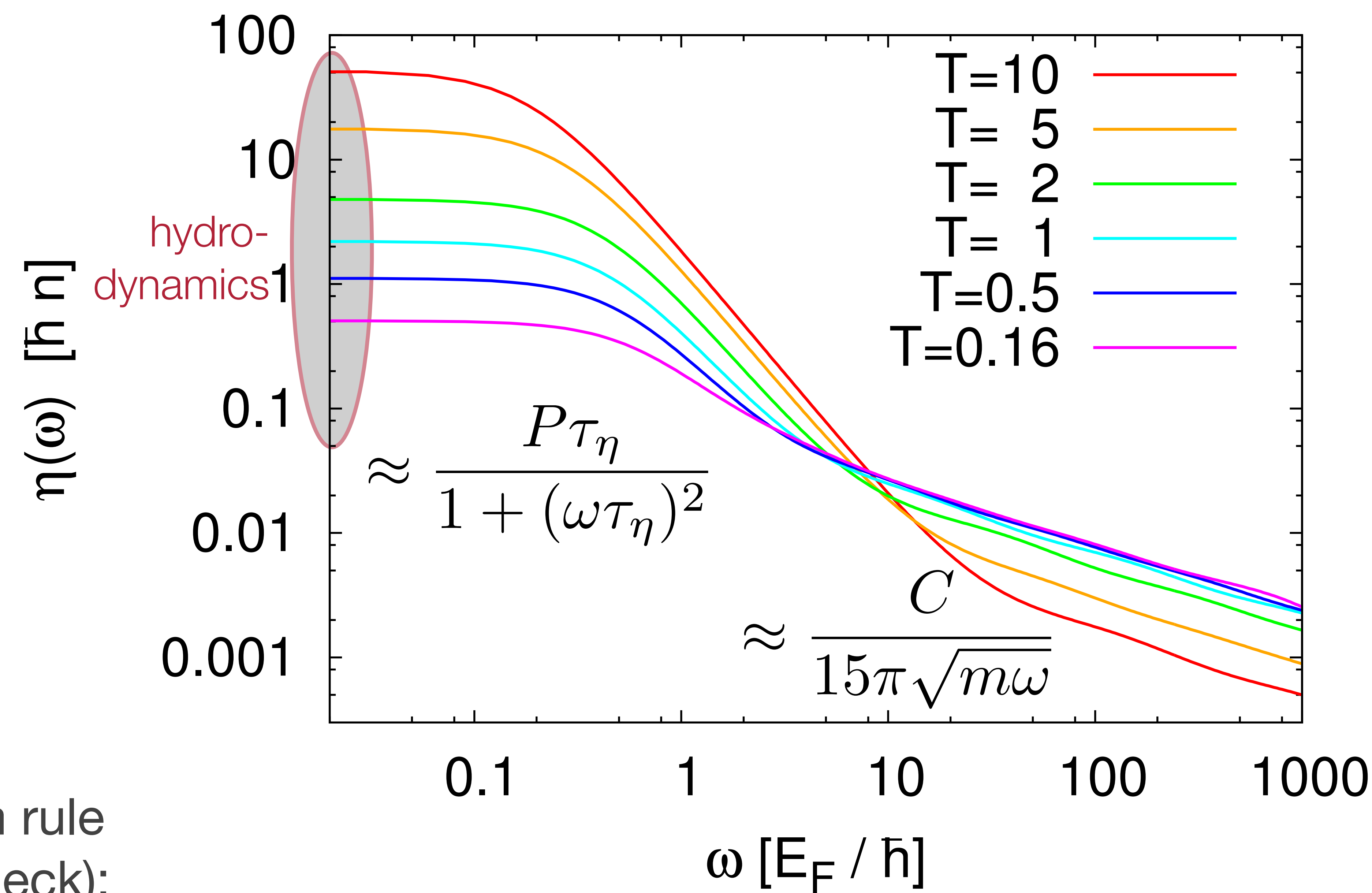
(resummed to infinite order)

- transport via fermions and bosonic molecules: very efficient description, satisfies conservation laws, scale invariance and Tan relations Enss PRA 2012

- assumes no quasiparticles: beyond Boltzmann kinetic theory, works near Tc

Dynamic shear viscosity

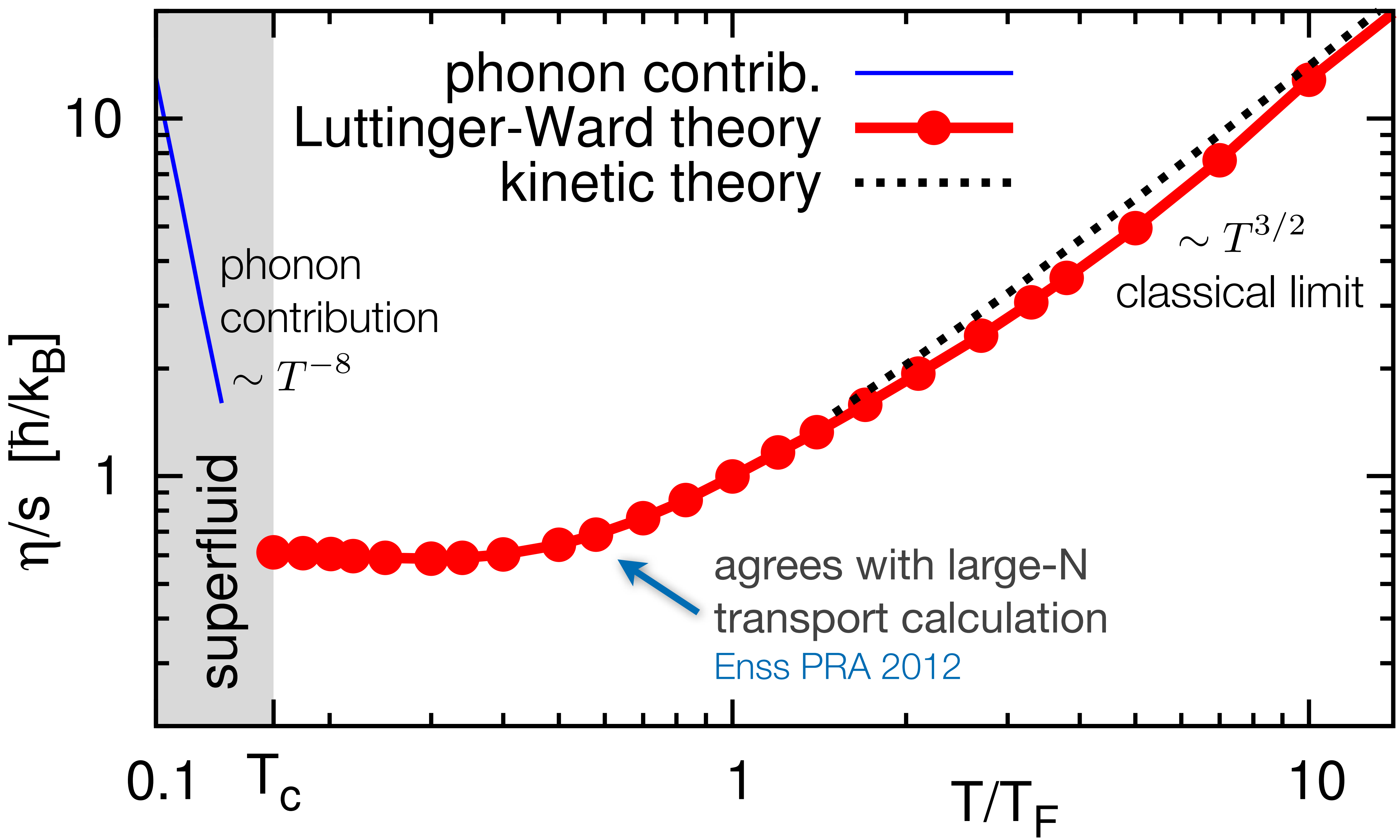
Enss, Haussmann & Zwerger 2011



exact viscosity sum rule
(nonperturbative check):

$$\frac{2}{\pi}\int_0^\infty d\omega\ [\eta(\omega) - \mathrm{tail}] = P - \frac{C}{4\pi m a}$$

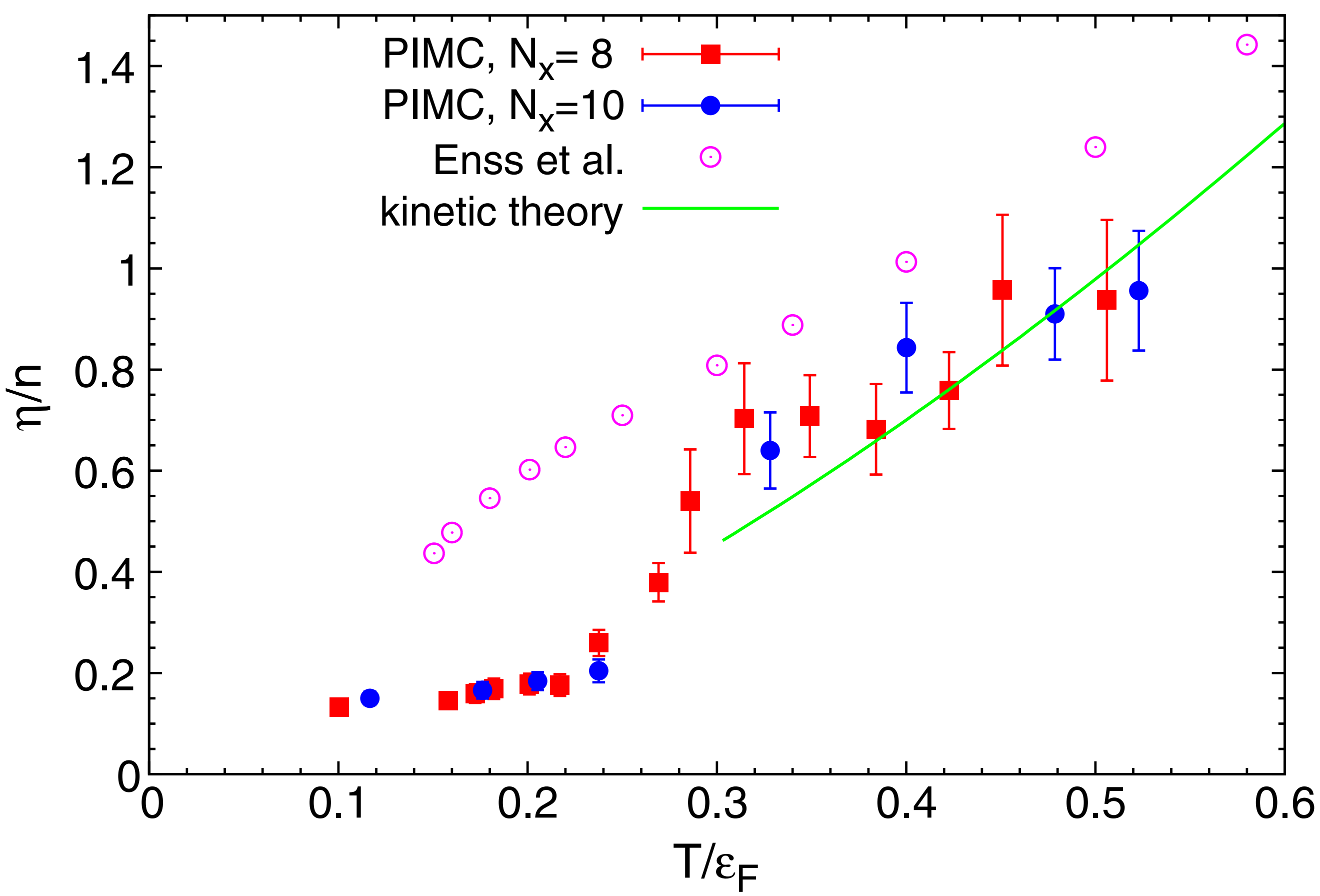
Enss, Haussmann & Zwerger 2011; Enss 2013; cf. Taylor & Randeria 2010

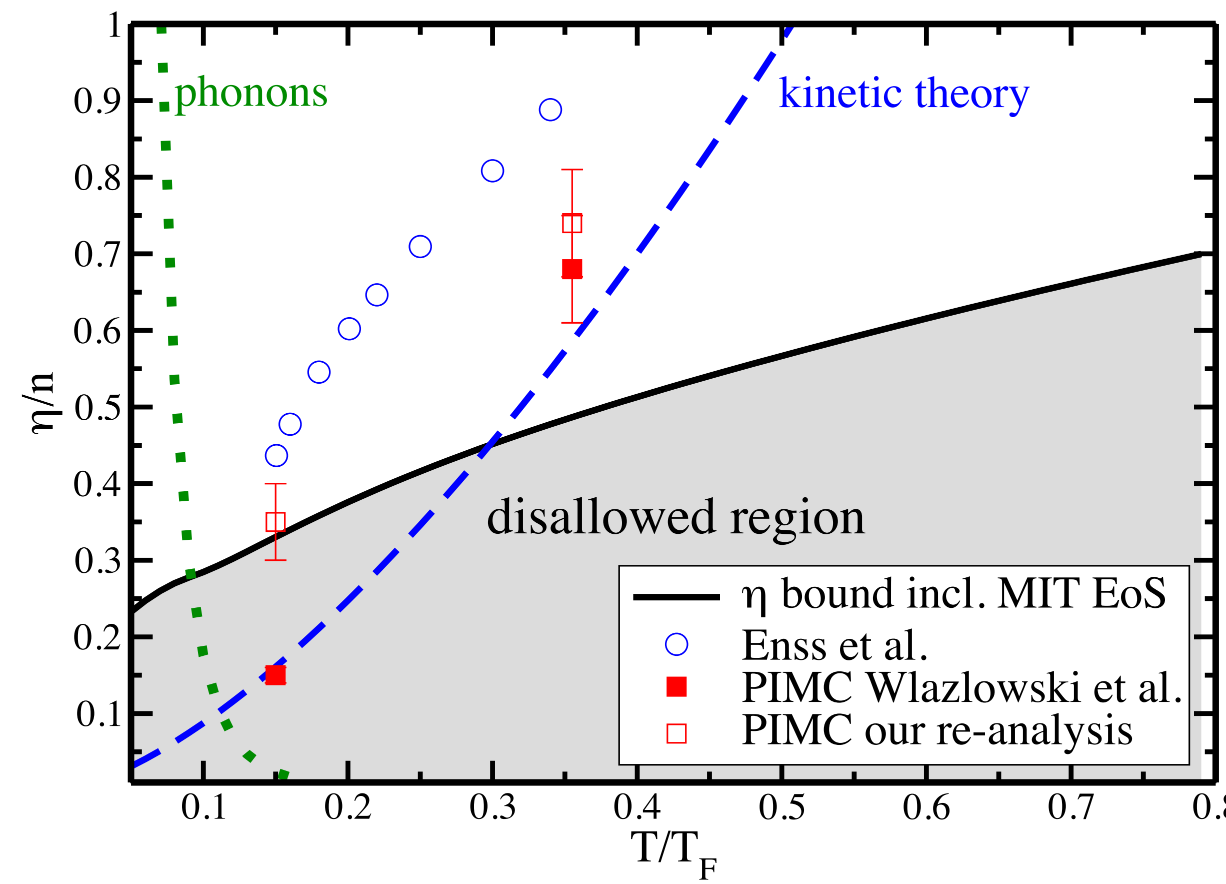


Shear viscosity/entropy of the unitary Fermi gas

Enss, Haussmann & Zwerger 2011

Shear viscosity bounds



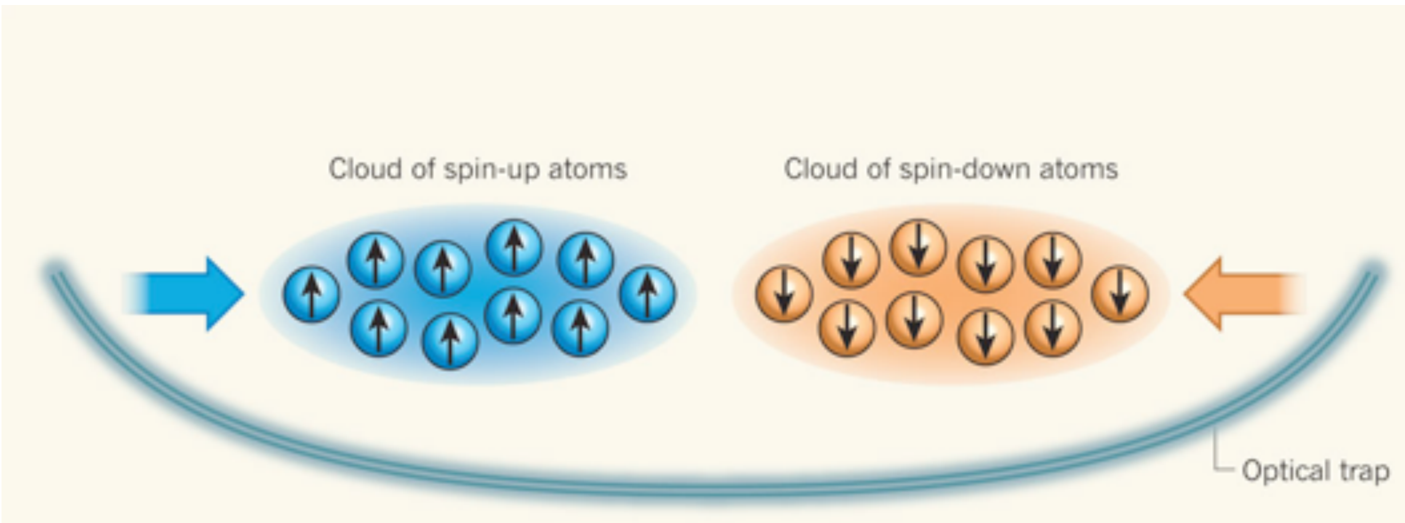


Wlazlowski et al. PRL 2012;
Wlazlowski et al. PRA 2013

Schäfer & Chafin PRA 2013;
Romatschke & Young PRA 2013

Spin transport with ultracold gases

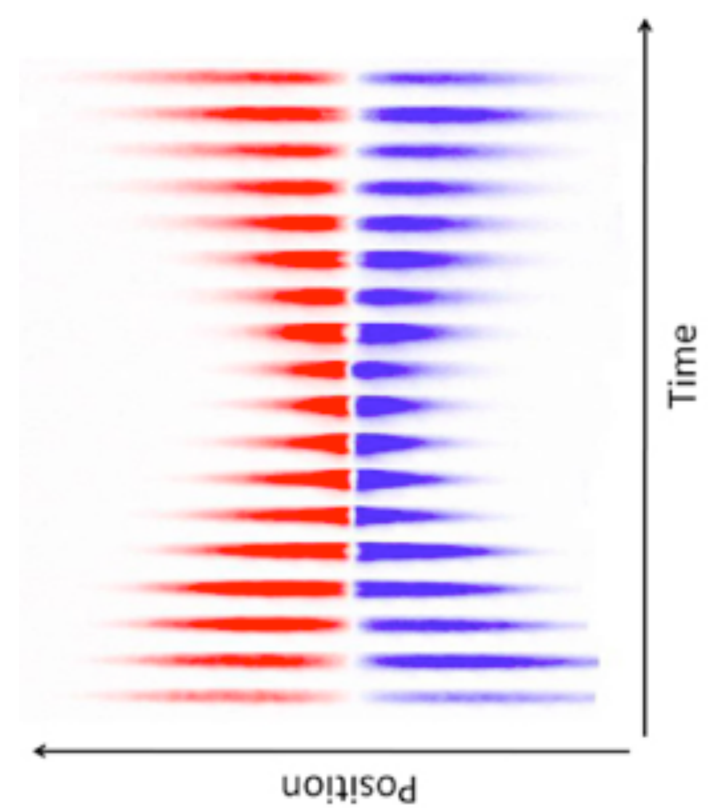
- experiment: spin-polarized clouds in harmonic trap



picture: J. Thomas 2011

- strongly interacting gas [movie courtesy Martin Zwierlein]:

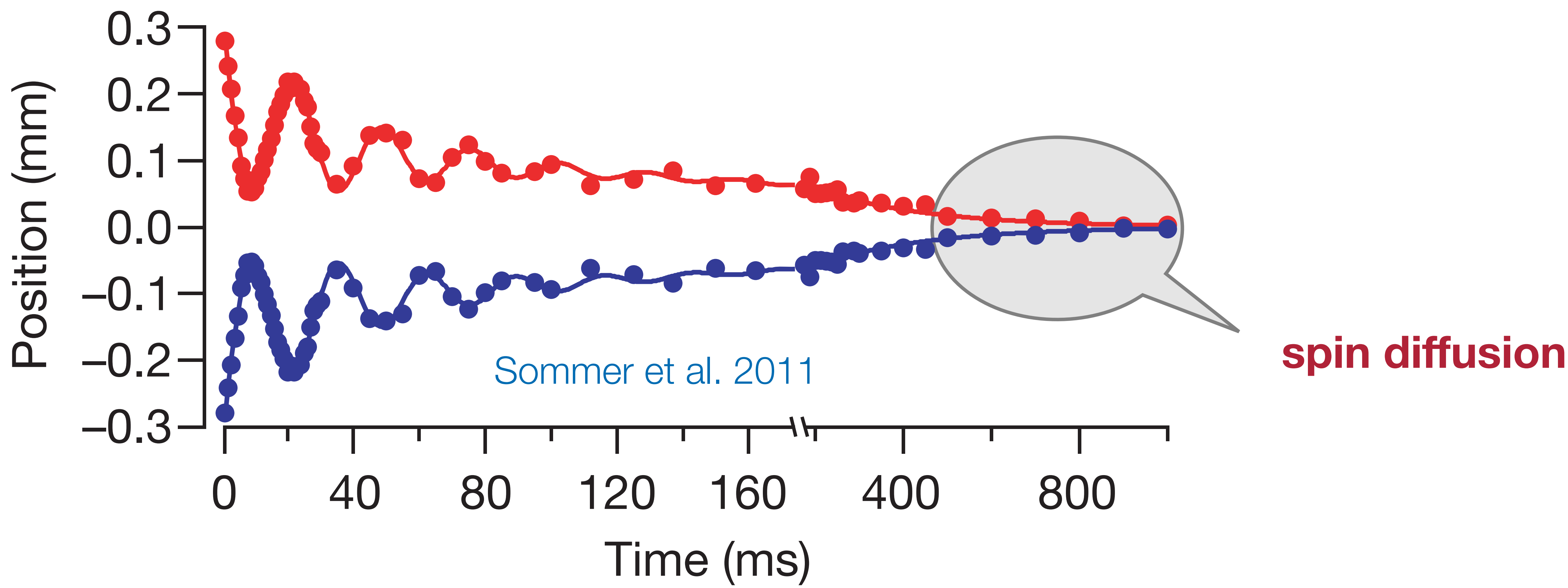
bounce!



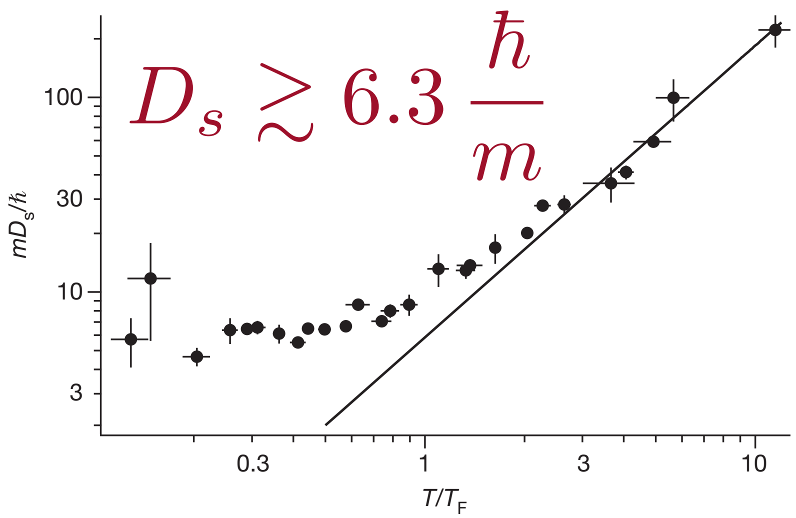
A.T. Sommer, M.J.H. Ku, G. Roati, M.W. Zwierlein, Nature 472, 201 (2011)

Spin diffusion

- scattering conserves total ↑+↓ momentum: mass current preserved
  but changes relative ↑-↓ momentum: spin current decays



Is there a quantum limit for diffusion?



Sommer et al. 2011

cf. spin Coulomb drag in GaAs quantum wells:

$$D_s \simeq 500\,\hbar/m$$

Weber et al. 2005

- **kinetic theory:** diffusion coefficient $D_s \approx v\ell_{\rm mfp}, \;\; v \simeq \hbar k_F/m, \;\; \ell_{\rm mfp} \gtrsim 1/k_F$

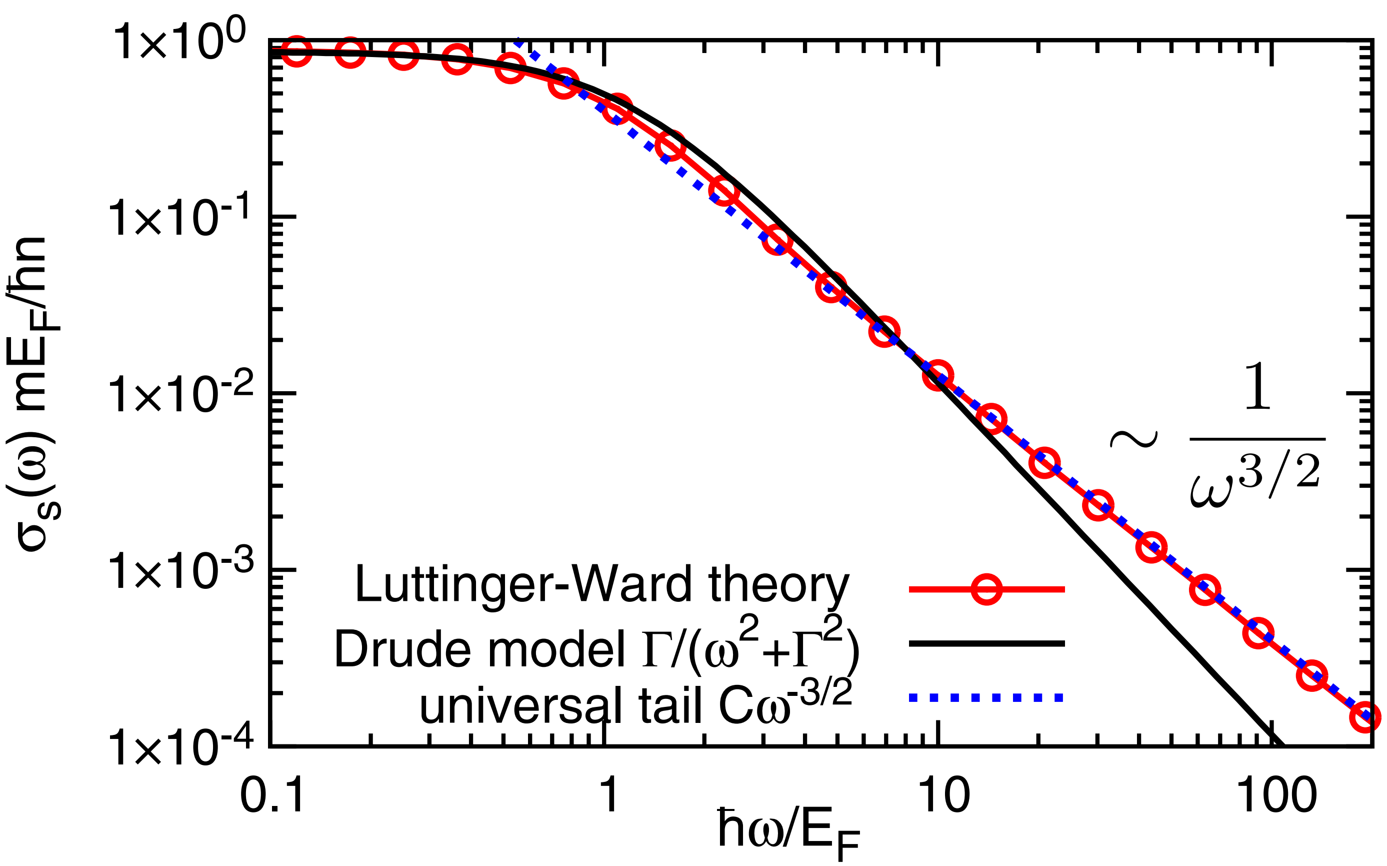
$$\boldsymbol{D_s \simeq \frac{\hbar}{m}}$$ **quantum limit of diffusion**

- spin conductivity from current correlations:

$$\sigma_s(\omega) = \frac{1}{\omega}\,\mathrm{Re}\int_0^\infty dt\, e^{i\omega t}\int d^3x\,\big\langle\big[j_s^z(\boldsymbol{x},t), j_s^z(0,0)\big]\big\rangle$$

with spin current operator $j_s(\boldsymbol{x},t) = j_\uparrow(\boldsymbol{x},t) - j_\downarrow(\boldsymbol{x},t)$

Dynamical spin conductivity



- exact high-frequency tail
  Hofmann PRA 2011;
  Enss & Haussmann PRL 2012

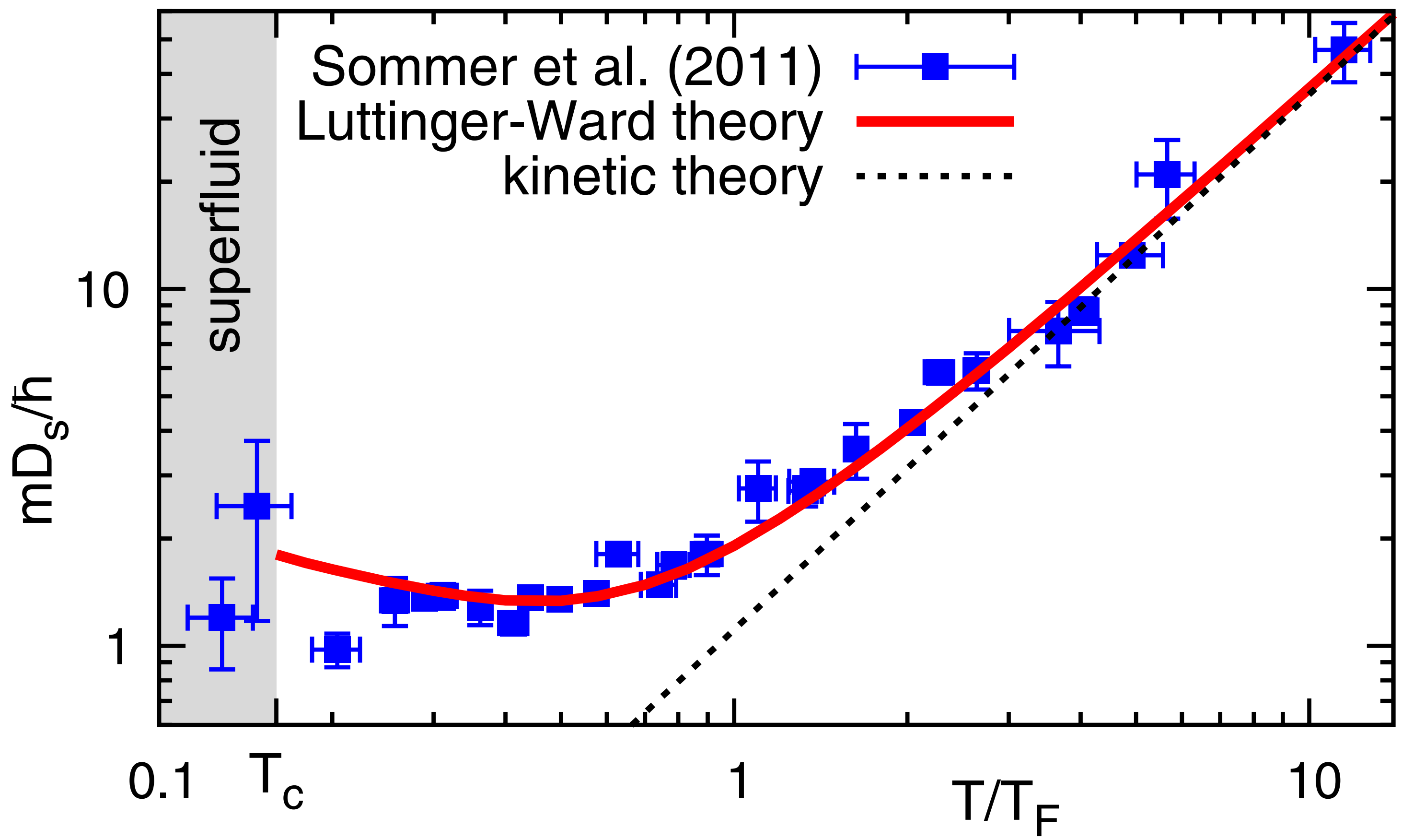
$$\sigma_s(\omega \to \infty) = \frac{C}{3\pi(m\omega)^{3/2}}$$

- satisfies spin sum rule despite tail in d<4 Enss, EPJ Spec.Topics 2013

$$\int \frac{d\omega}{\pi} \sigma_s(\omega) = \frac{n}{m}$$

Spin diffusivity

- obtain diffusivity from Einstein relation, $D_s = \dfrac{\sigma_s}{\chi_s}$



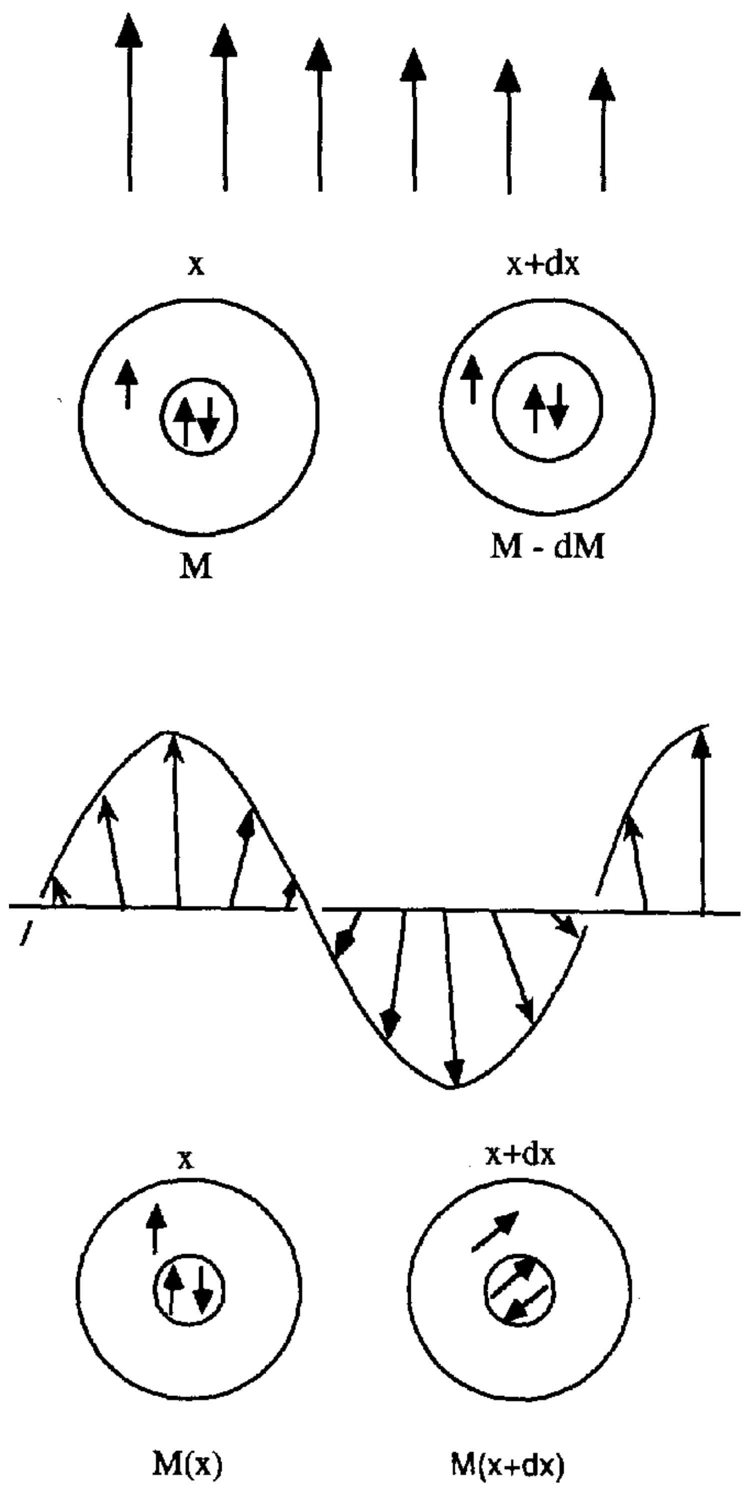
(experiment rescaled from trap to infinite homogeneous box)

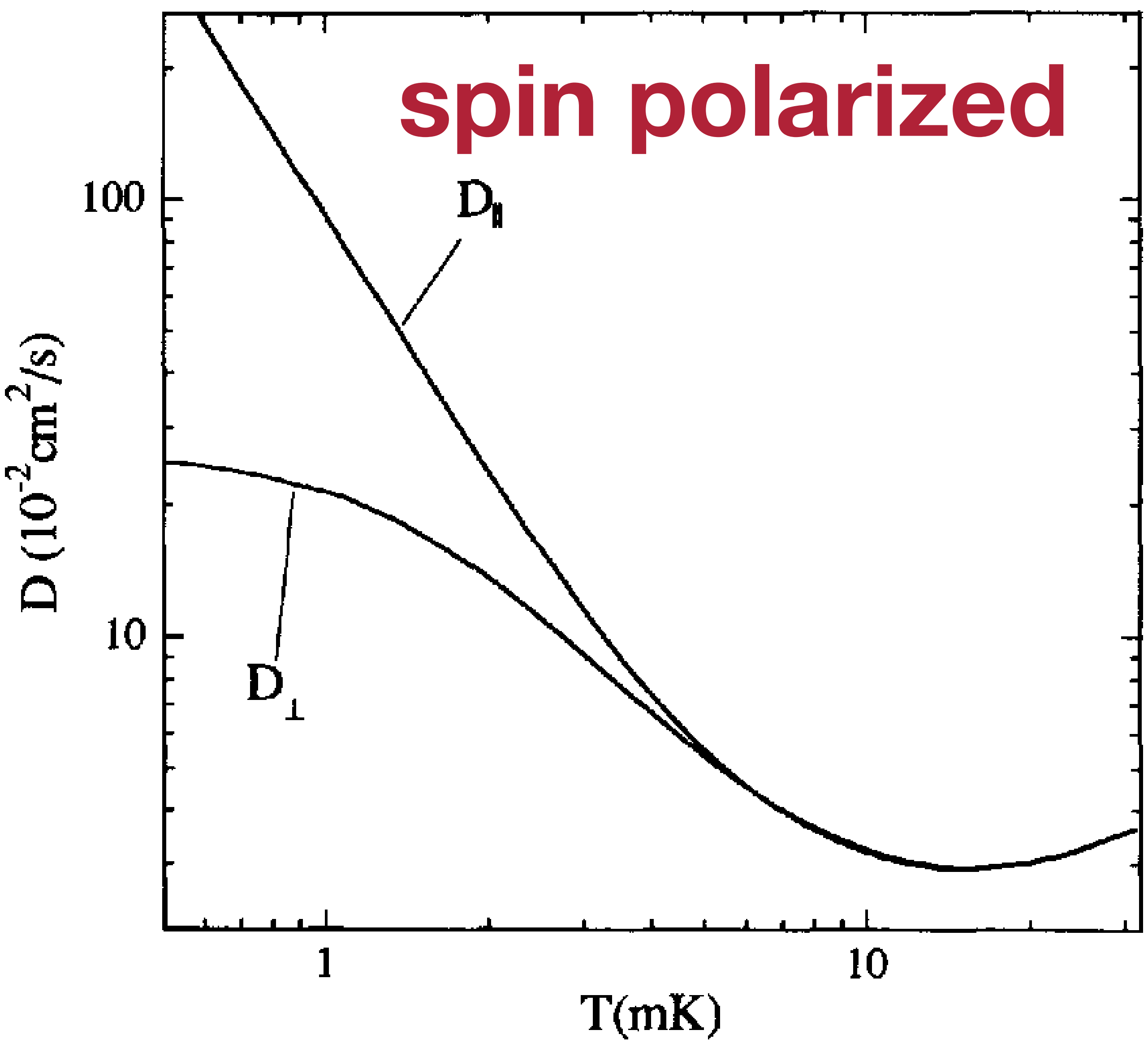
minimum $D_s \simeq 1.3\dfrac{\hbar}{m}$

Enss & Haussmann PRL 2012

- Quantum Monte Carlo simulation for finite lattice: $D_s \gtrsim 0.8\dfrac{\hbar}{m}$
  Wlazlowski et al. PRL 2013

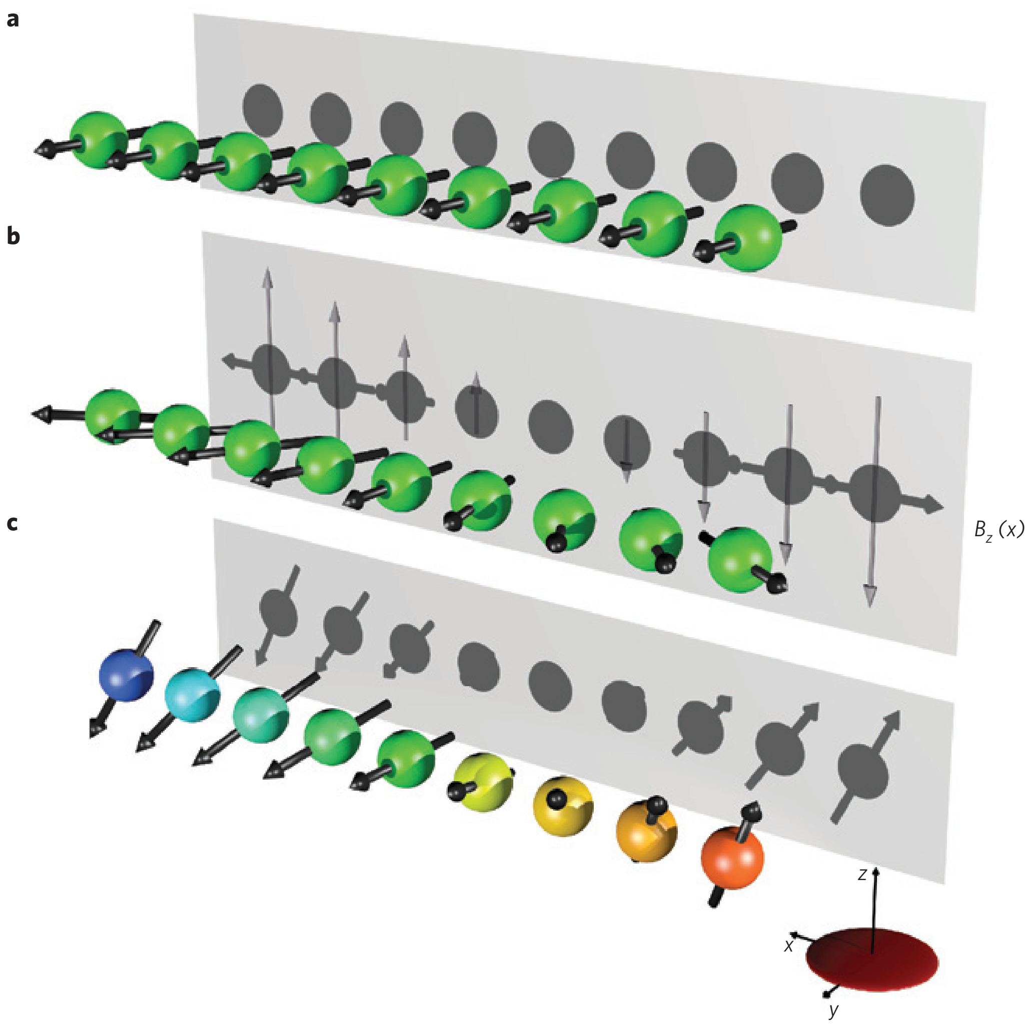
Longitudinal vs **transverse** spin diffusion

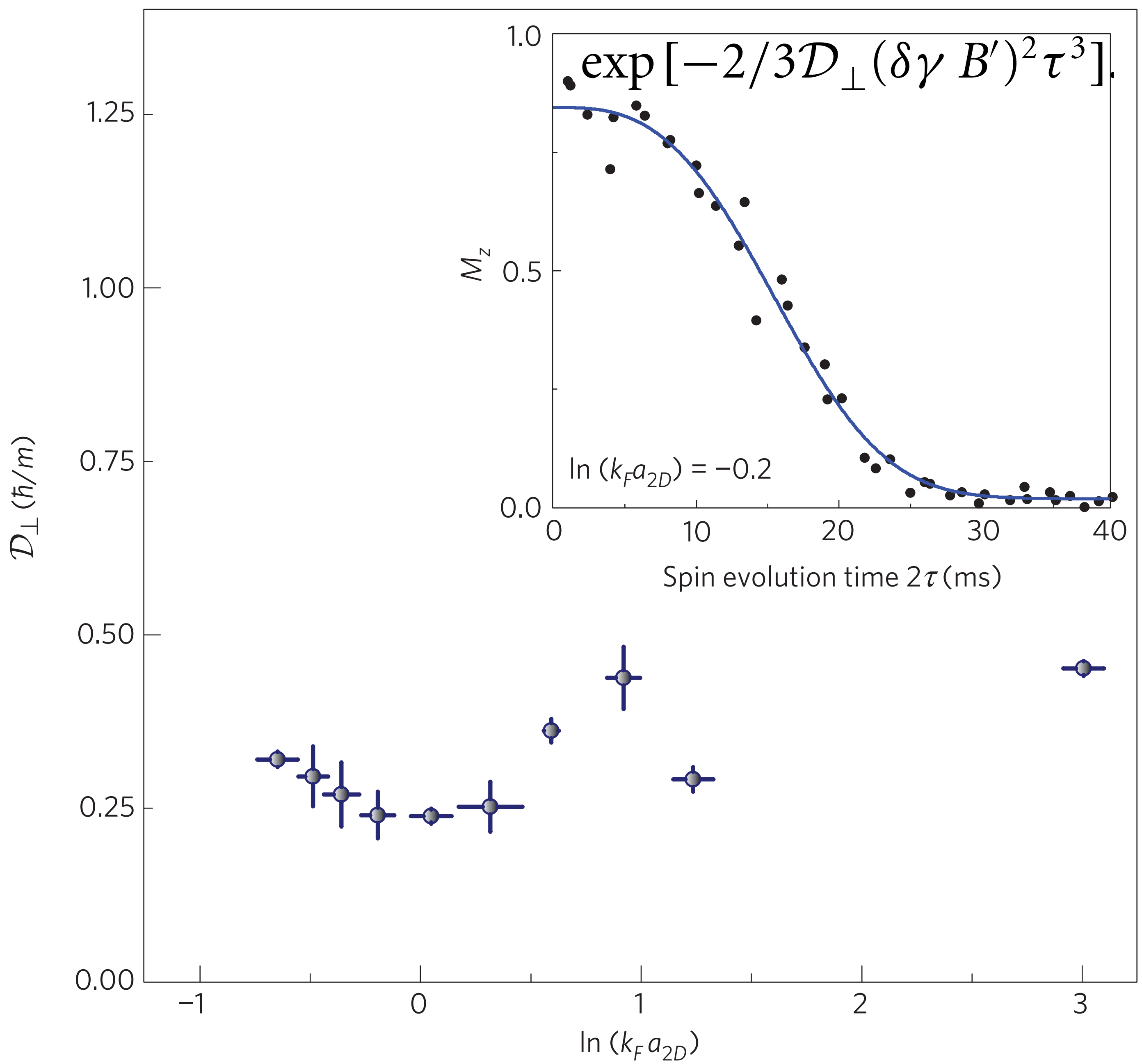




Mullin & Jeon 1992

Spin-echo experiment (Köhl group, Cambridge)





Koschorreck et al. Nature Physics 2013

Spin diffusion in kinetic theory

- local magnetization vector and gradient

$$\boldsymbol{\mathcal{M}}(\boldsymbol{r},t) = \mathcal{M}(\boldsymbol{r},t)\,\hat{\boldsymbol{e}}(\boldsymbol{r},t) \qquad \frac{\partial \boldsymbol{\mathcal{M}}}{\partial r_i} = \frac{\partial \mathcal{M}}{\partial r_i}\hat{\mathbf{e}} + \mathcal{M}\frac{\partial \hat{\mathbf{e}}}{\partial r_i}$$

- Boltzmann equation for spin distribution function

longitudinal

$$\frac{D\boldsymbol{\sigma}_p}{Dt} \equiv \frac{\partial \boldsymbol{\sigma}_p}{\partial t} - \sum_i v_{pi}\frac{\partial \mathcal{M}}{\partial r_i}\hat{\mathbf{e}}\sum_\sigma t_\sigma \frac{\partial n_{p\sigma}}{\partial \varepsilon_p}$$

$$+\sum_i v_{pi}\frac{\partial \hat{\mathbf{e}}}{\partial r_i}(n_{p+} - n_{p-}) + \boldsymbol{\Omega}\times\boldsymbol{\sigma}_p = \left(\frac{\partial \boldsymbol{\sigma}_p}{\partial t}\right)_{\mathrm{coll}}$$

Jeon & Mullin 1988;
Enss arXiv:1307.5175

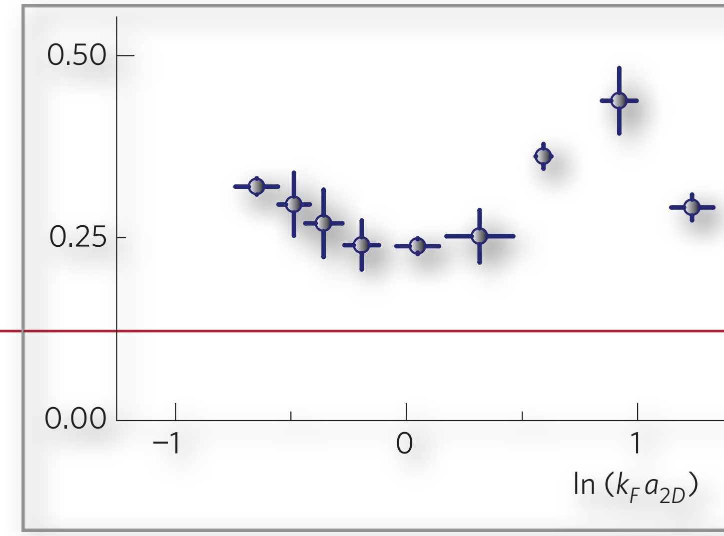
transverse **spin rotation**

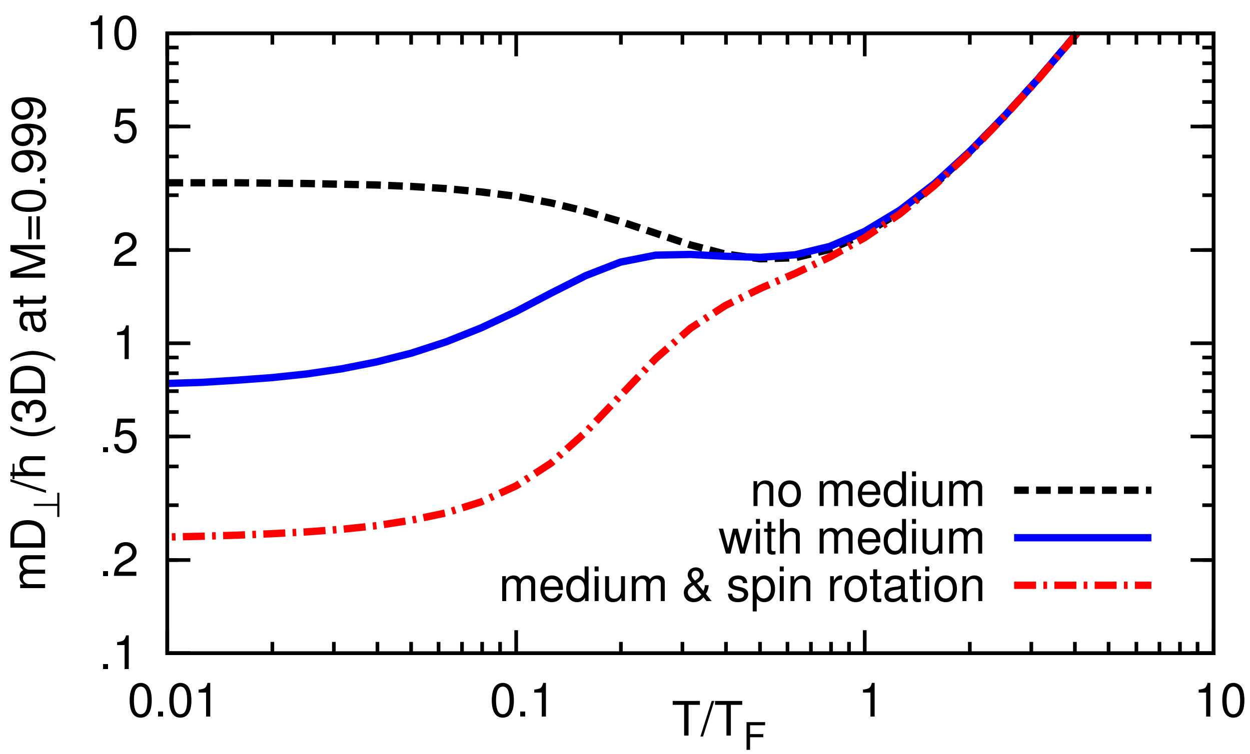
$$D_\perp = \frac{\alpha_\perp \tau_\perp}{1 + (\Omega_{\mathrm{mf}}\,\tau_\perp)^2}$$

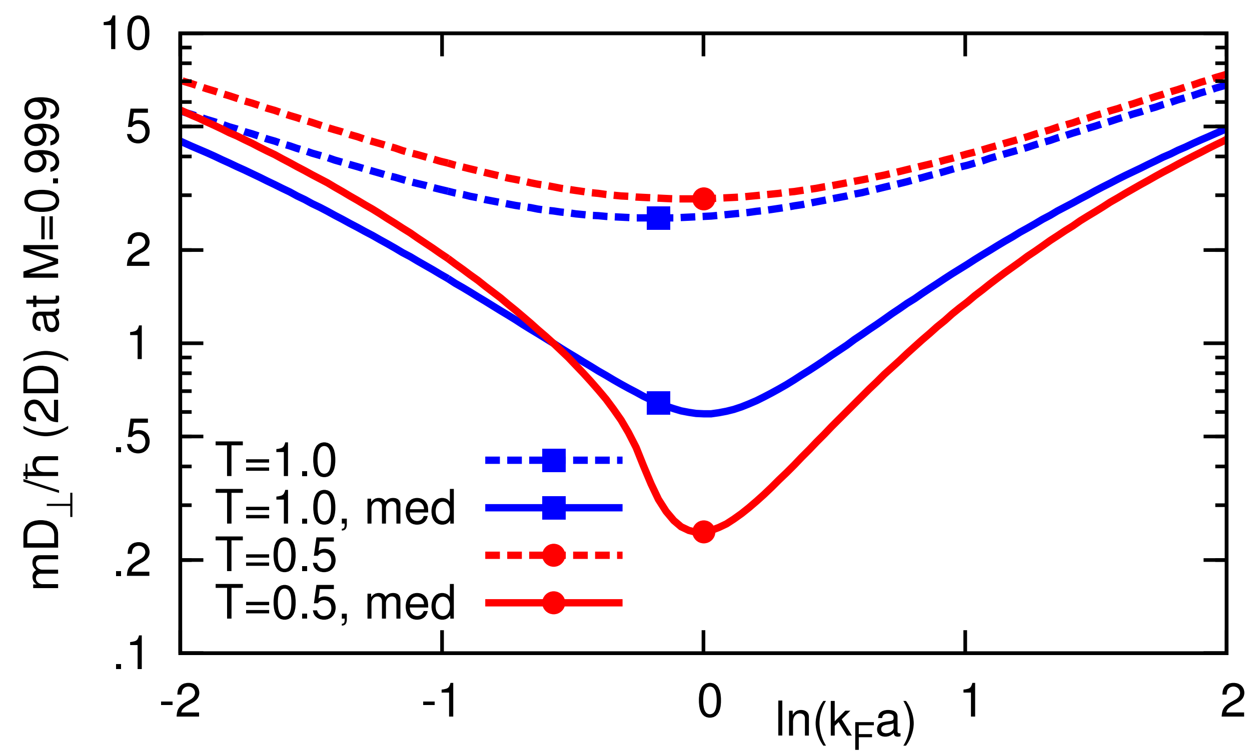
Leggett & Rice 1968;
Leggett 1970

- **many-body** T-matrix in collision integral and spin rotation Enss arXiv:1307.5175

Transverse spin diffusivity







3D - medium scattering and spin-rotation effect

2D - dependence on interaction and importance of medium effects
(cf. η: Enss, Küppersbusch & Fritz PRA 2012)

Enss arXiv:1307.5175

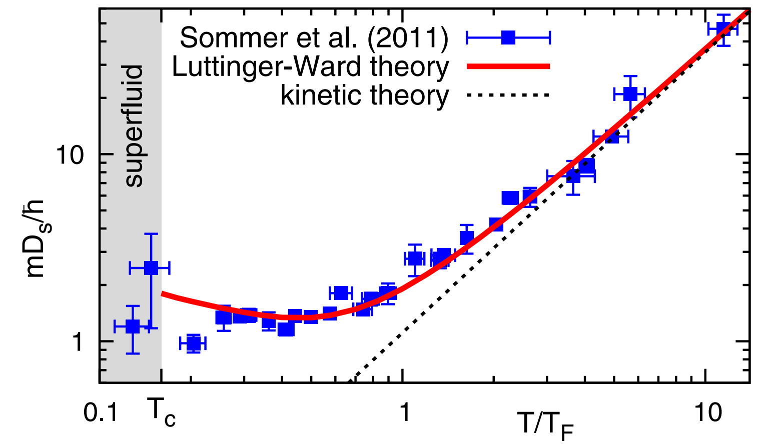
Conclusion and outlook

- strongly interacting Fermi gas with contact interaction:
  paradigm of many-body theory, precision experiments provide benchmark

- universal regime also at short distance $r_0 \lesssim r \lesssim \ell$: Tan contact density C

- mass transport (viscosity): less friction by strong int'n, nearly perfect fluidity
  Luttinger-Ward transport calculation (conserving, universal tail, works near Tc)

- longitudinal spin diffusion: $D_s \gtrsim 1.3\,\hbar/m$
  agrees quantitatively with MIT experiment



- transverse diffusion: slowest $D_\perp \approx 0.25\,\hbar/m$
  observed in polarized gas (strong medium effects at low T and large int'n)

- outlook: measure $D_\perp$ in 3D; Luttinger-Ward for polarized gas